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Inflation in Poincaré Gauge Gravitation Theory

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Exponential inflation and power-law inflation are studied in the Poincaré gauge gravitation theory for a Robertson–Walker metric.

I. In the early universe inflation is important because it can explain some problems such as homogeneity and flatness of the horizon (Guth, 1981). Inflation also can solve the problem of observed large-scale isotropy (Yuanjie, 1992). Recently, several authors have studied inflation in the Einstein–Cartan theory (Yuanjie *et al.*, 1993; Trautman, 1973; Hehl, 1980; Hayashi and Shirafuji, 1980). The main object of this paper is to point out that if the torsion contributions are included in the gravitation field equations according to the Poincaré gauge gravitation theory, both exponential inflation and power-law inflation can occur at a sufficiently early epoch, in which the torsion energy dominates the universe.

II. We start from the Robertson–Walker metric

$$ds^2 = -dt^2 + R(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin\theta \, d\psi^2\right) \tag{1}$$

where $R(t)$ is a scale factor.

Let $W^i = V^i_\mu dx^\mu$ and introduce vierbein fields V^i_μ

$$V^0_\mu = (1, 0, 0, 0), \qquad V^1_\mu = \left(0, \frac{R}{(1 - \kappa r^2)^{1/2}}, 0, 0\right)$$

$$V^2_\mu = (0, 0, Rr, 0), \qquad V^3_\mu = (0, 0, 0, Rr\sin\theta)$$

Then equation (1) becomes

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$$ds^2 = -(W^0)^2 + (W^1)^2 + (W^2)^2 + (W^3)^2 \tag{2}$$

Beakler (1983) showed that in the spherically symmetric case, the nonzero components of the torsion are

$$F^0_{01} = f(r, t), \qquad F^1_{10} = h(r, t)$$
$$F^2_{20} = F^3_{30} = k(r, t), \qquad F^2_{21} = F^3_{31} = -g(r, t) \tag{3}$$

Cartan's first equation, connecting the basis one-forms with the torsion F^k_{ij} can be written as

$$\frac{1}{2} F^i_{jk} W^j \wedge W^k = dW^i + W^i_j \wedge W^j \tag{4}$$

The nonzero components of the connection two-forms are

$$\begin{aligned}
W^0_1 &= W^1_0 = fW^0 + \left(h + \frac{\dot{R}}{R}\right)W^1, \\
W^3_0 &= -W^0_3 = \left(\kappa + \frac{\dot{R}}{R}\right)W^3 \\
W^0_2 &= W^2_0 = \left(\kappa + \frac{\dot{R}}{R}\right)W^2, \\
W^3_1 &= -W^1_3 = \left(\frac{(1 - \kappa r^2)^{1/2}}{Rr} - g\right)W^3 \\
W^2_1 &= -W^1_2 = \left(\frac{(1 - \kappa r^2)^{1/2}}{Rr} - g\right)W^2, \\
W^3_2 &= -W^2_3 = \frac{\mathrm{ctg}\,\theta}{Rr} W^3
\end{aligned} \tag{5}$$

Using the connection two-forms summarized above, we compute Cartan's second equation, which is

$$\frac{1}{2} R^i_{jkl} W^b \wedge W^i = dW^i_j + W^i_b \wedge W^k_j \tag{6}$$

We find the nonzero components of the curvature tensor

$$R^0_{101} = \frac{h\dot{R} + \dot{h}R + \ddot{R} - f(1 - \kappa r^2)^{1/2}}{R} \equiv A$$

$$R^2_{012} = R^3_{013} \frac{(1 - \kappa r^2)^{1/2}}{R\tau} \left(\kappa' r + \kappa + \frac{\dot{R}}{R}\right)$$

$$- \left(h + \frac{\dot{R}}{R}\right)\left(\frac{(1 - \kappa r^2)^{1/2}}{Rr} - g\right) \equiv D$$

$$R^2_{002} = R^3_{003} = \frac{R\dot{\kappa} + \dot{R}\kappa + \ddot{R}}{R} - f\left(\frac{(1 - \kappa r^2)^{1/2}}{R\tau} g\right) \equiv -C \tag{7}$$

$$R^2_{120} = R^3_{120} = \frac{R\dot{g} + g\dot{R}}{R} + f\left(\kappa + \frac{\dot{R}}{R}\right) \equiv -D$$

$$R^2_{121} = R^3_{131} = \frac{\kappa}{R^2} + \frac{g'r + g}{Rr}(1 - \kappa r^2)^{1/2} + \left(\kappa + \frac{\dot{R}}{R}\right)\left(h + \frac{\dot{R}}{R}\right) \equiv H$$

$$R^3_{223} = -\frac{\kappa}{R^2} + g^2 - \frac{2g(1 - \kappa r^2)^{1/2}}{Rr} - \left(\kappa + \frac{\dot{R}}{R}\right)^2 \equiv L$$

where overdot $= \partial/\partial t$ and prime $= \partial/\partial r$.

Using equation (7), we find the nonzero components of the Ricci tensor to be

$$\begin{aligned} &R_{00} = A - 2C, \qquad && R_{11} = -A - 2H \\ &R_{22} = R_{33} = C - H + L, \qquad && R_{01} = R_{10} = 2D \\ &\tilde{R} = -2A + 4C - 4H + 2L && \end{aligned} \tag{8}$$

where $\tilde{R}$ is the curvature scalar.

The nonzero components of the Einstein tensor are

$$\begin{aligned} &G_{00} = L - 2H, \qquad && G_{11} = -L - 2C \\ &G_{22} = G_{33} = A - C + H, \qquad && G_{01} = G_{10} = 2D \end{aligned} \tag{9}$$

We divide the total stress-energy into two parts:

1. The stress-energy tensor of matter

$$T_{ij} = V^\mu_i V^\nu_j T_{\mu\nu} \tag{10}$$

For a perfect fluid, we find easily

$$T_{00} = \rho, \qquad T_{11} = T_{22} = T_{33} = P \tag{11}$$

where ρ is an energy density and P is the pressure.

2. The stress-energy tensor of the gauge potential (Kibble, 1961)

$$\tau_{ij} = -\frac{1}{2} F^m_{ii} F^i_{jm} + \frac{1}{8} F^{mn}_i F^i_{mn} \eta_{ij} \tag{12}$$

where $\eta_{ij} = (-1, 1, 1, 1)$. Using equation (3), we find

$$\tau_{00} = \frac{f^2 - h^2}{4} - \frac{\kappa^2 + g^2}{2}$$
$$\tau_{11} = \frac{h^2 - f^2}{4} - \frac{\kappa^2 + g^2}{2} \qquad (13)$$
$$\tau_{22} = \tau_{33} = \frac{f^2 - h^2}{4}$$
$$\tau_{10} = \tau_{01} = \kappa g$$

The field equation is

$$G_{ij} = -8\pi T_{ij} - \lambda\tau_{ij} \qquad (14)$$

where λ is a coupling constant.

Because we have employed an isotropic and homogeneous universe model, we assume the torsions are only dependent on time. For simplicity, let $h = k, f = g = 0$. Thus the field equations become

$$L - 2H = -8\pi\rho - \frac{3\lambda}{4}\kappa^2 \qquad (15)$$

$$-L - 2C = -8\pi p - \frac{\lambda}{4}\kappa^2 \qquad (16)$$

$$A - C + H = -8\pi P - \frac{\lambda}{4}\kappa^2 \qquad (17)$$

$$D = 0 \qquad (18)$$

Substituting equation (7) into equations (15)–(18), we find

$$-\frac{3\kappa}{R^2} - 3\left(\kappa + \frac{\dot{R}}{R}\right)^2 = -8\pi\rho + \frac{3}{4}\lambda\kappa^2 \qquad (19)$$

$$\frac{\kappa}{R^2} + \left(\kappa + \frac{\dot{R}}{R}\right)^2 + 2\frac{R\dot{\kappa} + \dot{R}\kappa + \ddot{R}}{R} = -8\pi P + \frac{1}{4}\lambda\kappa^2 \qquad (20)$$

We assume that the energy of the matter and the energy of the torsion satisfy energy conservation, respectively; from the energy conservation of the torsion

$$\tau^{0i}_{ji} = 0 \qquad (21)$$

We find

$$\kappa^2 R^4 = C \quad \text{(const)} \tag{22}$$

From equation (22), we find

$$\frac{\dot{\kappa}}{\kappa} = -2\frac{\dot{R}}{R} \tag{23}$$

III.

1. Exponential Inflation

Let

$$R = R_0 e^{Dt} \tag{24}$$

where R_0 and D are constants.

Substituting (23) and (24) into (19) and (20), we find

$$\frac{8\pi\rho}{3} = \left(\frac{\lambda}{4} + 1\right)\kappa^2 + 2\kappa D^2 + D^2 + \frac{K}{R^2} \tag{25}$$

$$8\pi P = \left(\frac{\lambda}{4} - 1\right)\kappa^2 - 3D^2 - \frac{K}{R^2} \tag{26}$$

Letting $\lambda > 4$, we can make $R << 1$ in the very early stages of the universe, and thus $\kappa^2 >> K/R^2$; so that we may eventually have $P > 0$. If $K > 0$, from equation (25), we can see that $\rho > 0$ is naturally satisfied.

Through the above discussion, one finds that exponential inflation might occur if k^2 is large in the very early stages of the universe.

2. Power-Law Inflation

Let

$$R = R_0 t^m \tag{27}$$

where R_0 and m are constants.

Substituting (23) and (2) into (19) and (20), we find

$$\frac{8\pi\rho}{3} = \left(\frac{\lambda}{4} + 1\right)\kappa^2 + \frac{2m\kappa}{t} + \frac{K}{R^2} \tag{28}$$

$$8\pi P = \left(\frac{\lambda}{4} - 1\right)\kappa^2 - \frac{2m}{t^2} - \frac{3m^2}{t^2} - \frac{K}{R^2} \tag{29}$$

For $\lambda > 4$, it seems that the negative pressure can be removed from the model by the torsion term. So we may eventually have $P > 0$, and we easily see that we may have positive densities along with positive pressure.

Through the above discussion, one finds that the power-law inflation might occur if k^2 is large in the very early stages of the universe.

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